

## Free electron model – are you serious??

The following is a table of parameters for the “free electron model” for some simple metals. Let us do this exercise to realize how *un*-intuitive this model is! Pick Na. Estimate the inter-electron distance either from the electron concentration or the “radius parameter.” Then, calculate the Coulomb interaction energy between two electrons. Compare the Coulomb energy with the maximum kinetic energy (Fermi energy)! You need a calculator for this, or you can do a rough “back of the envelope” type calculation. The “Physical Constants” table should be useful.

**Table 1** Calculated free electron Fermi surface parameters for metals at room temperature

(Except for Na, K, Rb, Cs at 5 K and Li at 78 K)

Valency	Metal	Electron concentration, in $\text{cm}^{-3}$	Radius <sup>a</sup> parameter $r_n$	Fermi wavevector, in $\text{cm}^{-1}$	Fermi velocity, in $\text{cm s}^{-1}$	Fermi energy, in eV	Fermi temperature $T_F = \epsilon_F/k_B$ , in deg K
1	Li	$4.70 \times 10^{22}$	3.25	$1.11 \times 10^8$	$1.29 \times 10^8$	4.72	$5.48 \times 10^4$
	Na	2.65	3.93	0.92	1.07	3.23	3.75
	K	1.40	4.86	0.75	0.86	2.12	2.46
	Rb	1.15	5.20	0.70	0.81	1.85	2.15
	Cs	0.91	5.63	0.64	0.75	1.58	1.83
	Cu	8.45	2.67	1.36	1.57	7.00	8.12
	Ag	5.85	3.02	1.20	1.39	5.48	6.36
	Au	5.90	3.01	1.20	1.39	5.51	6.39
2	Be	24.2	1.88	1.93	2.23	14.14	16.41
	Mg	8.60	2.65	1.37	1.58	7.13	8.27
	Ca	4.60	3.27	1.11	1.28	4.68	5.43
	Sr	3.56	3.56	1.02	1.18	3.95	4.58
	Ba	3.20	3.69	0.98	1.13	3.65	4.24
	Zn	13.10	2.31	1.57	1.82	9.39	10.90
	Cd	9.28	2.59	1.40	1.62	7.46	8.66
3	Al	18.06	2.07	1.75	2.02	11.63	13.49
	Ga	15.30	2.19	1.65	1.91	10.35	12.01
	In	11.49	2.41	1.50	1.74	8.60	9.98
4	Pb	13.20	2.30	1.57	1.82	9.37	10.87
	Sn( <i>w</i> )	14.48	2.23	1.62	1.88	10.03	11.64

<sup>a</sup>The dimensionless radius parameter is defined as  $r_n = r_0/a_H$ , where  $a_H$  is the first Bohr radius and  $r_0$  is the radius of a sphere that contains one electron.